Principal Stress Separation in PhotoStress® Measurements

1.0 Introduction

In addition to its unique capability as a full-field technique for visualizing stress distribution, the PhotoStress® method provides quantitative stress measurement at any selected point or points on the coated surface of the test object. In common with conventional transmission photoelasticity, the basic measurement is ordinarily made with the light directed perpendicular to the surface of the photoelastic plastic (referred to as normal incidence). When made in this manner, the measurement yields the difference of principal strains in the coating.

At interior locations, removed from a free edge, the stress state is commonly biaxial; and it is sometimes necessary to determine the separate principal stresses, as well as their difference. In the past, the usual method of accomplishing this was to perform a second, independent, photoelastic measurement at the test point, using oblique-incidence along with the reflection polariscope.[1,2,3] Considerable skill is required, however, to make reasonably accurate measurements under oblique incidence. In addition to its other practical limitations, the oblique-incidence adapter is sometimes rendered inoperable due to mechanical interference with projecting features of the part surface.

This Tech Note describes a unique method of making the required additional measurement for determining the separate principal stresses from the photoelastically derived stress difference. The procedure uses a specially designed strain gage (stress-separator gage) which is applied to the coating surface after the normal-incidence reading has been made. Practical experience with the method demonstrates that it offers several advantages over oblique-incidence measurements. It is quick, easy to use, and it completely eliminates the need for highly developed photoelastic skills. In most cases, it is also more accurate than oblique-incidence determinations.

2.0 Measurements of $\sigma_1 - \sigma_2$ with PhotoStress

The basic relationship for strain measurement in a photoelastic coating can be expressed as follows:

$$\varepsilon_1 - \varepsilon_2 = N_n \lambda / 2kt \qquad (1)$$

Or,

$$\varepsilon_1 - \varepsilon_2 = N_n f \qquad (2)$$

Where:

$\varepsilon_1, \varepsilon_2$ = principal strains in coating
N_n = normal-incidence fringe order
λ = wavelength of yellow light (22.7μin, or 575 *nm*)
t = thickness of PhotoStress coating
k = strain-optic coefficient of coating
f = $\lambda/2kt$ = fringe value of coating

Assuming the strains in the coating precisely replicate those in the test-part surface, and assuming the part is stressed below its proportional limit, Hooke's law can be applied as follows to determine the difference of principal stresses:

$$\sigma_1 - \sigma_2 = \frac{E}{1+\nu}(\varepsilon_1 - \varepsilon_2) \qquad (3)$$

Where:

σ_1, σ_2 = principal stresses in test part
E = elastic modulus of test material
ν = Poisson's ratio of test material

The preceding relationships implicitly assume that the strains in the test part are unaffected by the presence of the bonded photoelastic coating, and that the strains in the coating are uniform through the coating thickness and equal to those in the surface of the test part. These assumptions are quite well satisfied for typical metal castings, forgings, and robust structural members, since the coating is much lower in elastic modulus, and is usually thin compared to the section depth of the test part. However, with low-modulus test materials and/or thin sections, correc-

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tions for reinforcement effects and nonuniform strain in the coating may be necessary. Procedures for making such corrections are given in a later section of this Tech Note.

3.0 Principal Stress Separation with Strain Gages

Although the resistance strain gage does not offer a practical alternative to PhotoStress as a means for full-field stress analysis, there are several ways in which the strain gage can serve as a very useful adjunct to the photoelastic coating method. One possibility, for example, is to use PhotoStress purely for the purpose of locating the most highly stressed points on the test part. A strain gage rosette can then be installed at each such point on an identical uncoated part (or on the same part, after removing the coating) to determine the complete strain state, from which the separate principal stresses can be calculated. This procedure is rarely practiced, however, since it discards the quantitative data carried by the photoelastic fringe pattern. It also creates uncertainties in the accuracy of the rosette locations.

An alternative method is to install conventional strain gage rosettes directly on the photoelastic coating at each point of interest. While conceptually simple, this technique involves a number of practical problems, and also introduces strain-measurement errors which cannot readily be quantified or corrected. The presence of the rosette, for instance, causes local reinforcement of the photoelastic coating, with the result that the strain indications tend to be too low. The degree of reinforcement is not definable, or subject to generalization, since the stiffness of the rosette varies with the gage construction, foil and backing thicknesses, grid and solder tab geometry, etc. Because the photoelastic plastic on which the gage is mounted is very low in thermal conductivity, there will also be problems with drift and instability of strain indication due to self-heating effects, unless extremely low gage-excitation voltage is employed. For many commercial strain indicators, this may be difficult to accomplish without introducing still other errors. These and similar problems arise primarily because neither the conventional rosette nor standard strain gage instrumentation is designed specifically for this highly specialized type of service. Perhaps of greatest importance in many practical cases would be the rather tedious task of reducing data from a number of strain gage rosettes, especially since the photoelastic analysis has already provided most of the necessary information.

The foregoing considerations suggest the possibility of developing a special-purpose strain gage, and circuitry, dedicated exclusively to the task of principal stress separation on PhotoStress coatings. With such an approach, the full capability of modern strain gage technology can be brought to bear in tailoring the gage parameters for optimum performance and ease of use in this single class of applications.

Gage construction details, for example, can be selected to minimize local reinforcement effects when installed on photoelastic plastic. Moreover, the residual reinforcement error can be entirely eliminated by calibrating the gage for its effective gage factor when installed on standard PhotoStress coatings. Other desirable characteristics such as pre attached leads can similarly be built into the design to temper the inaccuracies and application problems commonly associated with the use of strain gages on plastics. Finally, the grid can be specially configured to produce an output which greatly simplifies data reduction in calculating the separate principal stresses. All of the above features have been incorporated in a new special-purpose strain gage and instrument interface described in the following sections.

4.0 The PhotoStress Separator Gage

As noted earlier, a normal-incidence photoelastic measurement on the PhotoStress coating provides the difference in principal strains at the test point. If the sum of the principal strains can be measured at the same point, then the separate principal strains are obtainable by simply adding and subtracting the two measurements. It is evident from Mohr's circle of strain (Figure 1) that the center of the circle corresponds to $(\varepsilon_1 + \varepsilon_2)/2$, where ε_1 and ε_2 are the principal strains. However, the center also corresponds to $(\varepsilon_x + \varepsilon_y)/2$, where ε_x and ε_y are the normal strains in any two perpendicular directions. Thus, for any point P on the coating surface (Figure 2), the sum $\varepsilon_x + \varepsilon_y$ is constant, independent of the angle β, and equal to the sum $\varepsilon_1 + \varepsilon_2$. As a result, it is not necessary to measure the sum of the principal strains as such, but only the sum of any two perpendicular strains.

The PhotoStress Separator Gage is based on this fundamental principle of mechanics. As shown in Figure 3, the gage grid consists of two perpendicular elements (for sensing ε_x and ε_y), connected in series. The indicated strain from the gage then corresponds to $(\varepsilon_x + \varepsilon_y)/2$, and thus to $(\varepsilon_1 + \varepsilon_2)/2$, regardless of the gage orientation on the test surface. Representing the gage output signal by the symbol S_G, for convenience in algebraic manipulation,

$$S_G = \frac{\varepsilon_x + \varepsilon_y}{2} = \frac{\varepsilon_1 + \varepsilon_2}{2} \tag{4}$$

$$\text{And, } \varepsilon_1 + \varepsilon_2 = 2S_G \tag{5}$$

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Adding and subtracting with Eq. (2),

$$\frac{\begin{array}{l}\varepsilon_1 - \varepsilon_2 = N_n f \\ \varepsilon_1 + \varepsilon_2 = 2S_G\end{array}}{}\qquad \frac{\begin{array}{l}\varepsilon_1 - \varepsilon_2 = N_n f \\ \varepsilon_1 + \varepsilon_2 = 2S_G\end{array}}{}$$

$$\varepsilon_1 = S_G + \frac{N_n f}{2} \qquad (6) \qquad\qquad \varepsilon_2 = S_G - \frac{N_n f}{2} \qquad (7)$$

In practical applications, the usual procedure is to first complete all PhotoStress observations and normal-incidence measurements (N_n) on the coated test object. Following this, separator gages are installed on the coating at the potentially critical points established by PhotoStress analysis. Loads are then reapplied to the test object, and the separator gage measurements are recorded. Individual principal strains can then be calculated as shown in Equations (6) and (7).

The PhotoStress Separator Gage (manufactured by Vishay Micro-Measurements Division of Vishay Measurements Group, Inc.) embodies a number of special features designed for ease of use and optimum performance in PhotoStress applications. First in importance, of course, is that the gage does not require any particular angular orientation. It is simply bonded at the point where separation measurements are desired. Pre attached leadwires are provided to avoid the problems that users may have in soldering the leads to the gage before installation or, worse yet, attempting to do so after the gage is bonded to the photoelastic coating. The gage grid is also encapsulated in polyimide to eliminate the need for protective coating in most PhotoStress applications. Gage characteristics are summarized in the Table (on the following page), and basic specifications are given in the technical data sheet enclosed in the gage package. The specifications include the effective gage factor when the gage is installed on a PhotoStress coating — accounting for the slight local reinforcement of the coating by the gage. To allow for variations in the elastic modulus and thickness of the coating, the gage factor is given with a ±5% tolerance. Setting the gage factor control of the strain indicator to this value automatically compensates for the local reinforcement.

The PhotoStress Separator Gage is intended for use only with "high-modulus" coating materials (Types PS-1, PS-8, PL-1, PL-8). It should not be used with medium- or low-modulus coatings.

Table

Designation:	PSG-01-XX
Construction:	Constantan grid alloy, with polyimide backing and encapsulation.
Leadwires:	Preattached 6-in *(150-mm)*, insulated, flexible leadwires.
Grid Configuration:	Two perpendicular linear elements, series-connected.
Gage Length:	0.063 in *(1.60 mm)*.
Resistance:	200 ohms ±0.4%.
Effective Gage Factor:	2.05 ±5%*.
Self-Temperature-Compensation:	Compensated for steel or aluminum – PSG-01-06 and PSG-01-13, respectively.

* The effective gage factor information supplied with the PhotoStress Separator Gage is applicable only when the gage is installed on a PhotoStress coating bonded to a test part. Because of this and its other special characteristics, the gage is intended for use only as described in this Tech Note, and is not suitable for general-purpose strain measurement.

Grid resistance of the separator gage is 200 ohms; and it is intended that the gage be connected to an appropriate Vishay strain gage indicator through a specially designed interface module, the Model 330 (Figure 4). The interface module is a four-channel switch-and-balance unit containing precision resistive circuits for attenuating gage excitation voltage and supplying appropriate bridge completion for the 200-ohm gage. The resulting gage current is approximately 1 mA, yielding a power level well below the threshold at which self-heating effects could destabilize the strain indication.

The Model 330 Interface Module attenuates the gage excitation voltage, and the gage output signal, by a factor of five. Designating the bridge output signal as R_i,

$$R_i = \frac{S_G}{5} = \frac{\varepsilon_1 + \varepsilon_2}{10} \qquad (8)$$

Thus, with the multiplier switch of the P-3500 set in the X1 position, if the unit digit of the display is interpreted as $10\mu\varepsilon$, instead of $1\mu\varepsilon$, the displayed strain equals the sum of the principal strains $(\varepsilon_1 + \varepsilon_2)$ directly. Letting Σ_ε represent the displayed signal (with this interpretation),

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$$\textstyle\sum_\varepsilon = 10R_i = 2S_G = \varepsilon_1 + \varepsilon_2 \quad (9)$$

Substituting into Eqs. (6) and (7),

$$\varepsilon_1 = \frac{\sum_\varepsilon + N_n f}{2} \quad (10)$$

$$\varepsilon_2 = \frac{\sum_\varepsilon - N_n f}{2} \quad (11)$$

5.0 Corrections for Coating Effects

As noted in the preceding section, the reinforcement of the photoelastic coating by the strain gage is automatically compensated for when the gage factor control of the strain indicator is set at the value specified in the technical data sheet accompanying the PhotoStress Separator Gage. Consideration must still be given, however, to possible reinforcement effects of the coating on the test object; and, in the case of bending applications, to strain-extrapolation effects (nonuniform strain through the coating thickness).

When the test object is low in elastic modulus, or the test section is thin, a sensible fraction of the applied load may be borne by the coating, which thus acts as a reinforcement on the member. If this condition exists, strains in the test object are smaller (for a given applied load) with the photoelastic coating present than they would be without it. The resulting reinforcement error occurs both in the photoelastically measured principal strain difference ($\varepsilon_1 - \varepsilon_2$) and in the sum of the principal strains ($\varepsilon_1 + \varepsilon_2$) as measured by the PhotoStress Separator Gage.

For thin members in bending, not only is there reinforcement of the object (since the coating increases the section modulus), but the strain at the outer surface of the coating is necessarily greater than that at the test-part surface. This occurs because the strain increases linearly with distance from the neutral axis about which bending takes place. With these conditions, the observed photoelastic fringe order in the coating corresponds to the average strain through the coating thickness; i.e., to the strain at the mid-thickness. Similarly, the strain sensed by the PhotoStress Separator Gage is too great by the strain increment through the total coating thickness. This effect is referred to as a strain-extrapolation error, since the bending-induced strain gradient normal to the part surface is extrapolated outward through the coating.

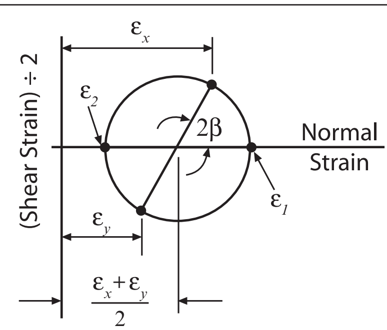

Figure 1. Mohr's circle of strain, illustrating the equivalence of $(\varepsilon_x + \varepsilon_y)/2$ and $(\varepsilon_1 + \varepsilon_2)/2$.

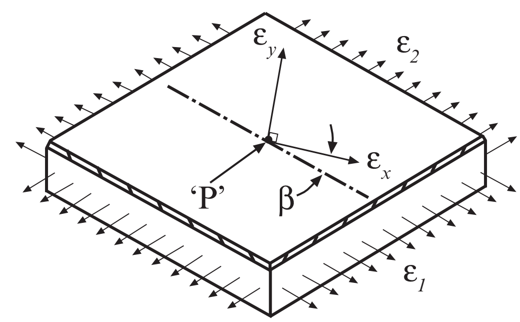

Figure 2. Coating surface at a selected test point, defining ε_x, ε_y, ε_1, ε_2.

Figure 3. PhotoStress Separator Gage (see Table).

Figure 4. Model 330.

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In summary, when the elastic modulus of the test material is low, or the section thickness is small, corrections to both the photoelastic fringe order and the strain indicator reading may be required. These two sets of corrections are described separately in the following subsections.

A. Photoelastic Corrections

Well-established relationships are available to permit simple correction of measured fringe orders in either of two basic modes of loading: plane stress (flat objects with in-plane loads, thin-walled pressure vessels, etc.), and pure bending.[4,5] The correction for plane stress can be expressed as:

$$C_{PS} = 1 + E^* t^* \tag{12}$$

Where:

C_{PS} = factor by which the observed fringe order in plane stress must be multiplied to obtain the corrected fringe order.

$E^* = \frac{E_C}{E_S}$ = ratio of the elastic modulus of the photoelastic coating to that of the specimen.

$t^* = \frac{t_C}{t_S}$ = ratio of the coating thickness to the specimen thickness.

The corresponding correction factor for applied bending moment is:

$$C_B = \frac{1 + E^*\left(4t^* + 6t^{*2} + 4t^{*3}\right) + E^{*2}\, t^{*4}}{1 + t^*} \tag{13}$$

Where:

C_B = factor by which the observed fringe order in bending must be multiplied to obtain the corrected fringe order.

E*, t* = elastic-modulus and thickness ratios as defined for Eq. (12).

Equations (12) and (13) apply specifically to cases of plane stress and pure bending, respectively. (See TN-706 for the appropriate correction factor for different materials.) Correction methods for mixed plane stress and bending are described in References 4 and 5.

B. Strain Gage Corrections

When reinforcement and strain-extrapolation errors are significant in the photoelastic measurements, they are also significant in the strains indicated by the PhotoStress Separator Gage. Separate correction factors have again been developed for the plane-stress and bending cases.

In plane stress, the correction factor for the indicated strain is the same as that for the fringe order; namely,

$$K_{PS} = C_{PS} = 1 + E^* t^* \tag{14}$$

Where:

K_{PS} = factor by which the indicated strain in plane stress is multiplied to obtain the corrected strain.

The correction factor for applied bending moment differs somewhat from the corresponding photoelastic correction because the measurement is made at the surface of the coating instead of its mid-thickness:

$$K_B = \frac{1 + E^*\left(4t^* + 6t^{*2} + 4t^{*3}\right) + E^{*2}\, t^{*4}}{1 + 2t^* + E^* t^{*2}} \tag{15}$$

Correction is made, in either case, by multiplying the indicated strain (Σ_ε) by the appropriate factor. Equations (14) and (15) have been plotted in Figures 5A and 5B to permit simple correction of the strain measurements. These figures apply to the epoxy family and polycarbonate photoelastic coatings, respectively. Equations (14) and (15) should be used when the elastic modulus of the coating differs from the values used in Figures. 5A and 5B.

After correcting both the measured fringe order and the indicated strain, the adjusted values of N_n and Σ_ε are substituted into Eqs. (10) and (11) to obtain the separate principal strains. The separate principal stresses are then calculated from the biaxial Hooke's law as follows:

$$\sigma_1 = \frac{E_S}{1 - \nu_S^2}\left(\varepsilon_1 + \nu_s \varepsilon_2\right) \tag{16}$$

$$\sigma_2 = \frac{E_S}{1 - \nu_S^2}\left(\varepsilon_2 + \nu_s \varepsilon_1\right) \tag{17}$$

Where:

σ_1, σ_2 = principal stresses

E_S, ν_S = elastic modulus and Poisson's ratio of test material

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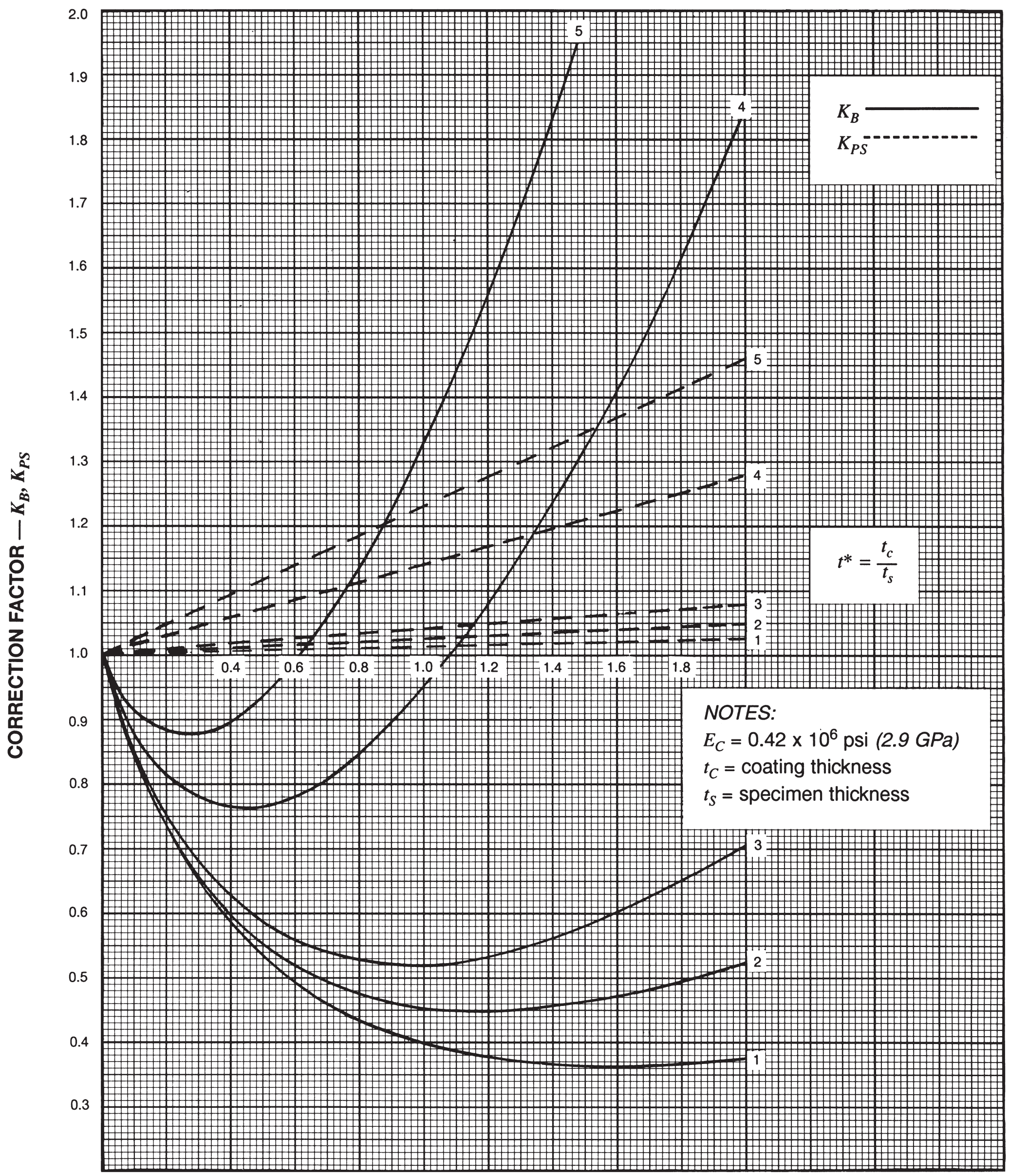

KEY	Specimen Elastic Modulus, E_S	
	in 10^6 psi	in GPa
1. Steel	30	207
2. Cast Iron	16	110
3. Aluminum	10	69
4. Reinforced Plastic	3.0	21
5. Wood	1.8	12.5

Figure 5a. Correction factor K_B (bending) and K_{PS} (plane stress) for separator gage used with photoelastic epoxy coating Types PS-8, PL-1, and PL-8. Multiply the measured reading by the appropriate correction factor to obtain the corrected measurement.

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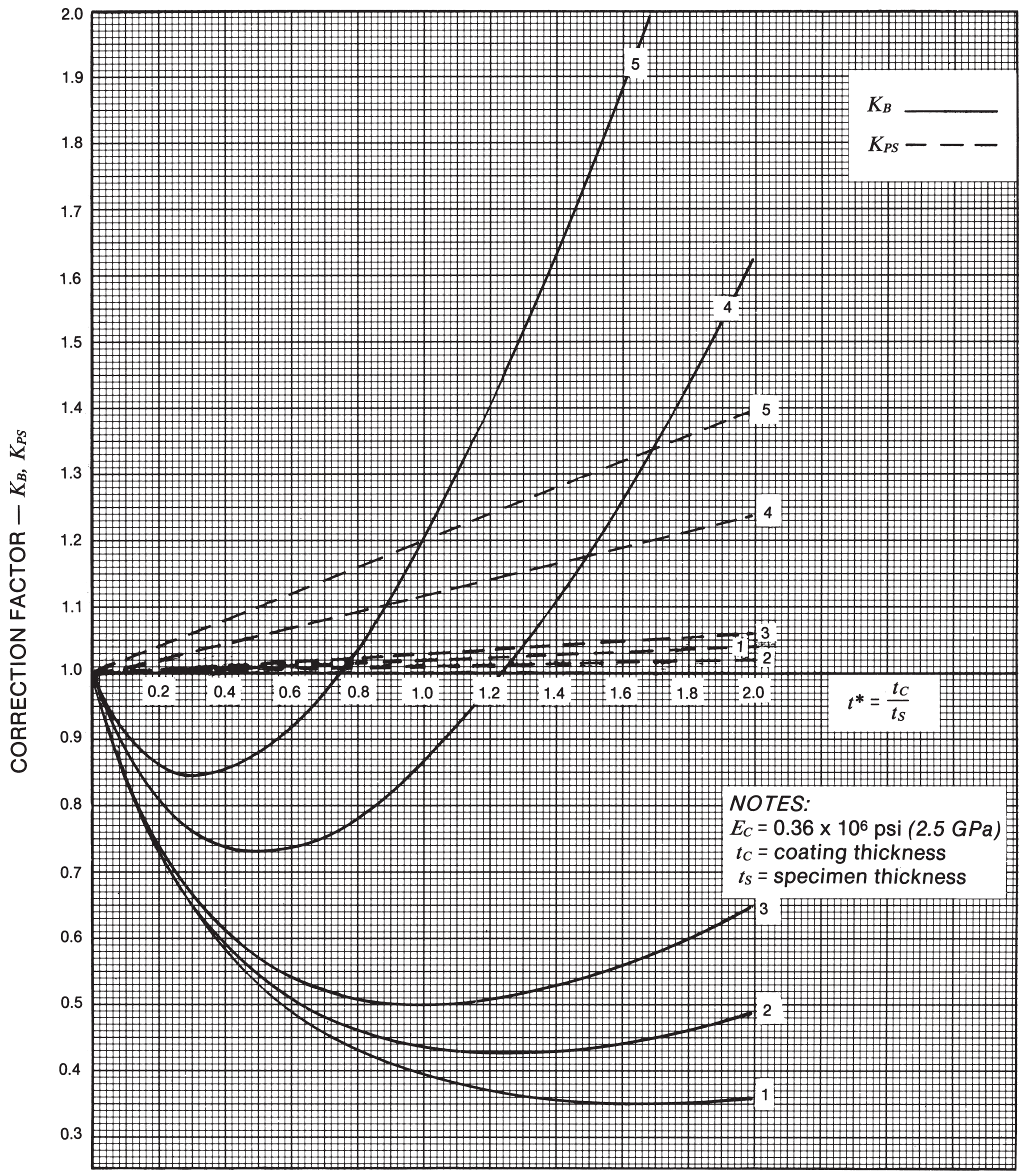

KEY	Specimen Elastic Modulus, E_S	
	in 10^6 psi	*in GPa*
1. Steel	30	207
2. Cast Iron	16	110
3. Aluminum	10	69
4. Reinforced Plastic	3.0	21
5. Wood	1.8	12.5

Figure 5a. Correction factor K_B (bending) and K_{PS} (plane stress) for separator gage used with photoelastic polycarbonate coating Type PS-1. Multiply the measured reading by the appropriate correction factor to obtain the corrected measurement.

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C. Imposed Deformation Loading

Some structural components are loaded, or deformed, to prescribed shapes by imposing in-plane (plane stress) or out-of-plane (bending or flexural) deformations. When the imposed deformations are in-plane, the strain field is effectively the same in the coated and uncoated parts and no corrections to experimental measurements are required. However, when out-of-plane deformations are imposed (e.g., bending to a predetermined curvature), special correction factors $C_{B\Delta}$ and $K_{B\Delta}$ must be used instead of C_B and K_B as defined in Eqs. (13) and (14).

The correction factors for imposed deformation are:

$$C_{B\Delta} = \frac{1 + E^* t^*}{1 + t^*} \qquad (18)$$

And by similar analysis:

$$K_{B\Delta} = \frac{1 + E^* t^*}{1 + 2t^* + E^* t^{*2}} \qquad (19)$$

Where:

E*, t* = elastic modulus and thickness ratios as defined for Eq. (12).

Values of $C_{B\Delta}$, per Eq. (18), are plotted in Figure 5 of TN-706.

Separator Gage correction factors $K_{B\Delta}$, per Eq. (19), are plotted in Figure 6 on page 7 and for practical engineering applications the $K_{B\Delta}$ for the epoxy family and polycarbonate coatings can be considered as equal. Calculation shows a worst case difference of only 3% for rigid vinyl and approximately a 1½% difference for magnesium. No attempt is made in Figure 6 to reflect these small variations. Equations (18) and (19) should be used when the elastic modulus of the coating differs from the values used in Figure 5 of TN-706 and Figure 6 herein.

6.0 Numerical Examples

The following numerical examples are provided to illustrate the data-reduction procedures for calculating the separate principal stresses from the combined photoelastic and separator strain gage measurements. The first example is representative of typical applications where there is no perceptible reinforcement of the test object by the PhotoStress coating. Two further examples show how to correct the measured photoelastic and strain gage data when reinforcement and strain-extrapolation effects are present.

Example 1

Assume that portions of a heavy steel structural member have been coated with Type PL-1 photoelastic plastic, 0.075 in (1.9 mm) thick. The fringe value (f) for the coating is 1513$\mu\varepsilon$/fringe. Using the current LF/Z model reflection polariscope, the normal-incidence measurement at a point of interest on the coating yields a reading of 2.1 fringes (N_n). The load is then removed from the member, and a PhotoStress Separator Gage is installed on the coating at the same point. Gage orientation is arbitrary, since the sum of any two perpendicular strains is equal to the sum of the principal strains.

The strain gage is connected to a portable strain indicator through the Model 330 Interface Module and the instrument is balanced to zero indication for the no-load condition. For example, with the multiplier switch of a P-3500 set to X1, the same load is reapplied to the structural member, after which the indicated strain (Σ_ε = 10 times the display reading) is 520$\mu\varepsilon$. In this case, it is not necessary to correct either N_n or Σ_ε for reinforcement or strain-extrapolation errors, since the effect of the coating on the elastic response of the structure is negligible. Substituting f, N_n, and Σ_ε into Eqs. (10) and (11),

$$\varepsilon_1 = \frac{520 + 2.1 \times 1513}{2} = 1849\mu\varepsilon$$

$$\varepsilon_2 = \frac{520 - 2.1 \times 1513}{2} = -1329\mu\varepsilon$$

These principal strains are then substituted into the biaxial Hooke's law to determine the principal stresses:

$$\sigma_1 = \frac{E_s}{1 - \nu_S^2}(\varepsilon_1 + \nu_s \varepsilon_2)$$

With E_S = 30.0 × 10^6 psi (*207 GPa*), and ν_S = 0.29,

$$\sigma_1 = \frac{30 \times 10^6}{1 - (0.29)^2}(1849 - 0.29 \times 1329) \times 10^{-6}$$

And,

$$\sigma_1 = 47\ 940 \text{ psi } (330\ MPa)$$

$$\sigma_2 = \frac{30 \times 10^6}{1 - (0.29)^2}(-1329 + 0.29 \times 1849) \times 10^{-6}$$

And,

$$\sigma_2 = -25\ 970 \text{ psi } (-179\ MPa)$$

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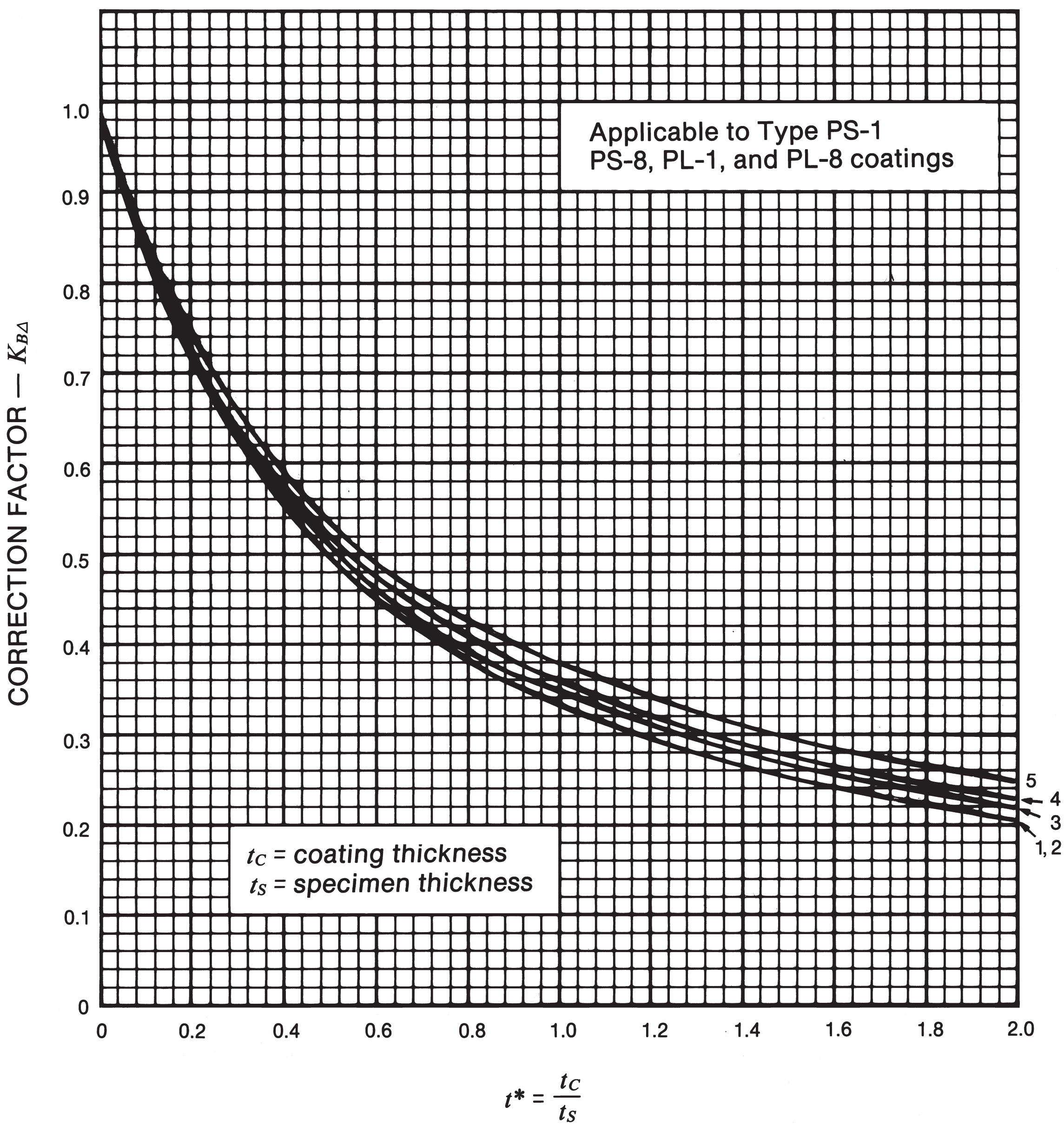

Figure 6. Correction factor KB (imposed curvature) for Separator Gage. Multiply the Separator Gage strain measurement by the appropriate factor to obtain the correct strain. Refer to KEY in Figure 5 B for Specimen Elastic Modulus, E_s.

Example 2

Assume, for this example, a flat aluminum-alloy specimen 0.125 in (*3.18 mm*) thick, coated with PL-1 photoelastic plastic 0.090 in (*2.3 mm*) thick. The fringe value of the coating is 1261$\mu\varepsilon$/fringe. With purely in-plane loads applied to the specimen (plane stress), the uncorrected normal-incidence photoelastic measurement (N_n) at the test point is 1.5 fringe. Following the procedure described in Example 1, a PhotoStress Separator Gage is installed on the coating at the same point, and the uncorrected indicated strain (Σ_ε), after reapplying the load, is +800$\mu\varepsilon$.

Since it can be suspected in this instance that the coating may sensibly reinforce the test specimen, the photoelastic

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and strain gage measurements should be corrected using the plane-stress correction factors C_{PS} and K_{PS}. For this purpose, it is first necessary to calculate the thickness ratio, t^*:

$$t^* = \frac{t_c}{t_s} = \frac{0.090}{0.125} = 0.72$$

Then, from Figure 3A of TN-706, the correction factor C_{PS} for the observed fringe order is 1.03, representing a modest 3% reinforcement effect. Since $K_{PS} = C_{PS}$, the same correction factor applies to the indicated strain from the PhotoStress Separator Gage as well.

Thus,

$$N_n = \hat{N}_n \times 1.03 = 1.5 \times 1.03 = 1.55 \text{ fringe}$$
$$\sum_\varepsilon = \hat{\sum}_\varepsilon \times 1.03 = 800 \times 1.03 = 824\mu\varepsilon$$

where:

$\hat{N}_n, \hat{\sum}_\varepsilon$ = observed (uncorrected) values of the variables.

Calculating the principal strains from Eqs. (10) and (11),

$$\varepsilon_1 = \frac{824 + 1.55 \times 1261}{2} = 1389\mu\varepsilon$$

$$\varepsilon_2 = \frac{824 - 1.55 \times 1261}{2} = -565\mu\varepsilon$$

And, substituting into Eqs. (16) and (17), with $E_S = 10 \times 10^6$ psi (*69 GPa*) and $\nu_S = 0.32$, the principal stresses are:

$$\sigma_1 = \frac{10 \times 10^6}{1 - (0.32)^2}(1389 - 0.32 \times 565) \times 10^{-6}$$

And,

$$\sigma_1 = 13\ 460 \text{ psi } (93\ MPa)$$

$$\sigma_2 = \frac{10 \times 10^6}{1 - (0.32)^2}(-565 + 0.32 \times 1389) \times 10^{-6}$$

And,

$$\sigma_2 = -1340 \text{ psi } (-9.2\ MPa)$$

Example 3

To permit ready comparison between coating effects in plane stress and those in bending, this example assumes the same specimen as in Example 2, with all of the same parameters, except that the specimen is subjected to pure bending. The observed photoelastic and strain gage measurements are also assumed the same.

With $t^* = 0.72$, reference to Figure 3A of TN-706 gives the bending correction factor C_B for the fringe order as 0.76. The corrected fringe order is then:

$$N_n = \hat{N}_n \times C_B = 1.5 \times 0.76 = 1.14 \text{ fringe}$$

Similarly, the bending correction factor K_B for the strain indication is read from Figure 5A as 0.54, from which:

$$\sum_\varepsilon = \hat{\sum}_\varepsilon \times K_B = 800 \times 0.54 = 432\mu\varepsilon$$

The principal strains are:

$$\varepsilon_1 = \frac{432 + 1.14 \times 1261}{2} = 935\mu\varepsilon$$

$$\varepsilon_2 = \frac{432 - 1.14 \times 1261}{2} = -503\mu\varepsilon$$

And the corresponding principal stresses become:

$$\sigma_1 = \frac{10 \times 10^6}{1 - (0.32)^2}(935 - 0.32 \times 503) \times 10^{-6}$$

And,

$$\sigma_1 = 8620 \text{ psi } (59\ MPa)$$

$$\sigma_2 = \frac{10 \times 10^6}{1 - (0.32)^2}(-503 + 0.32 \times 935) \times 10^{-6}$$

And,

$$\sigma_2 = -2270 \text{ psi } (-15.6\ MPa)$$

A comparison of Example 3 with Example 2 illustrates that, for this case, the coating effect in bending is much greater than in plane stress. That this is a common situation for low-modulus or thin test objects can be judged from the nature of the correction-factor curves in Figure 3 of TN-706 and Figure 5 herein. It is also evident from these examples that the error can be quite large if not corrected.

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